

Special relativity as the limit of an Aristotelian universal friction theory under Reye's assumption

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Abstract

This work explores a classical mechanical theory under two further assumptions: (a) there is a universal dry friction force (Aristotelian mechanics), and (b) the variation of the mass of a body due to wear is proportional to the work done by the friction force on the body (Reye's hypothesis). It is shown that mass depends on velocity as in Special Relativity, and that the velocity is constant for a particular characteristic value. In the limit of vanishing friction the theory satisfies a relativity principle as bodies do not decelerate and, therefore, the absolute frame becomes unobservable. However, the limit theory is not Newtonian mechanics, with its Galilei group symmetry, but rather Special Relativity. This result suggests to regard Special Relativity as the limit of a theory presenting universal friction and exchange of mass-energy with a reservoir (vacuum). Thus, quite surprisingly, Special Relativity follows from the absolute space (ether) concept and could have been discovered following studies of Aristotelian mechanics and friction. We end the work confronting the full theory with observations. It predicts the Hubble law through tired light, and hence it is incompatible with supernova light curves unless both mechanisms of tired light (locally) and universe expansion (non-locally) are at work. It also nicely accounts for some challenging numerical coincidences involving phenomena under low acceleration.

1 Introduction

The Galilean principle of relativity establishes that the mechanical laws are the same for a family of observers in uniform relative motion: the so called inertial observers. This important principle, generalized by Einstein to all the physical laws, provided, together with the principle of constancy of the speed of light, the logical foundation of the special theory of relativity.

Already before this remarkable accomplishment, the relativity principle was regarded as fundamental as it meant a radical departure from the old Aristotelian physics according to which a body, not acted upon by a force, would stay at rest in a privileged absolute frame. To recognize the principle of relativity means to recognize that there is no absolute space and that uniform motion has to be understood as *relative* to another observer.

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Most modern physicists and philosophers would regard Aristotelian physics as quite naive. It seems that Aristotle misinterpreted the tendency of objects of coming at rest (with respect to the earth) as the consequence of a general principle according to which a body tends to its *natural state*, instead as the result of friction. According to this school of thought the main merit of Galilean physics is that of identifying the natural state of a body in absence of friction as the fundamental one. This state is that of uniform motion.

However, this is not a fair historical reconstruction. As Dugas [7] points out, Aristotle was very well aware that probably, in vacuum, a body would have moved in uniform motion indefinitely. However, from that he inferred that absolute vacuum is impossible. In other words, it is not because Aristotle did not recognize the role of friction that he did not come to the principle of relativity, but rather, because he regarded the principle that every body should come to absolute rest as more fundamental than the relativity principle! Indeed, some kind of deceleration is necessary in order to give physical observability to the concept of absolute space which Aristotle was not prepared to abandon.

Despite this historical clarification, we can safely regard the relativity principle as a cornerstone of modern physics, as we owe to it the full development of Newtonian mechanics and the discovery of Special Relativity. Nevertheless, the idea of cosmological flow and the discovery of the CMB radiation suggest that, perhaps, there is indeed a privileged reference frame and that, after all, some elements of Aristotle's absolute space could still reenter into play.

The aim of this work is twofold. First we will challenge most naive criticisms to the absolute space idea, showing that, in fact, some aspects of relativity theory, like the relativistic mass formula, are a natural consequence of Aristotelian mechanics. This rather puzzling result will be of interest for the philosopher and the physicist alike. Indeed, not even Galilean physics, which embodies the relativity principle can claim a similar prediction (as it misses the invariance of the speed of light). This result is based on an assumption concerning the way in which friction alters the mass (energy) content of a body. This is Reye's assumption, an old relation from nineteenth century applied mechanics which is promoted here to a general principle. As we shall see, the role of the speed of light will be played by a characteristic velocity which turns out to be insensitive to friction. A body with this speed would indefinitely preserve it. On the contrary, bodies having subluminal speeds would decrease their velocity reaching a status of absolute rest. The just mentioned results will be independent of several details concerning the universal friction force acting on bodies.

As a second objective we will explore the consequences of the simple model of Coulomb (dry) friction. We will show that it predicts the Pioneer anomaly and the Hubble law, where the latter is explained through a kind of universal tired light mechanism. More importantly, the model naturally explains the observed coincidence between the Hubble constant and the Pioneer anomalous acceleration.

Then, we show that the friction force can be given a perpendicular component which is able to reproduce several aspects of MOND theory, including the mechanism explaining the flatness of the rotational curves of galaxies, the Tully-Fisher law, and the value of the coefficient in the Tully-Fisher law. In other words, this model seems to be able to explain many odd phenomena that have been observed for accelerations

smaller or of the order of a certain critical value a_p although it is not able to explain other observations like the supernova light curves. Still, it could suggest some important ideas towards the resolution of some puzzles of modern physics.

2 Aristotelian mechanics

Before we embark in these developments it will be convenient to clarify what we mean by Aristotelian mechanics. There is a certain consensus among historians that Aristotle had developed a *Physics* rather than a *Mechanics*. His physics included principles that would be hardly considered as mechanical by a modern physicist but that allowed him to infer some mechanical consequences. Certainly, he had not in mind a simple axiomatic structure as in the posterior Newtonian mechanics. Despite that, there are repeated statements in his treatises which can be converted into quantitative mechanical laws [6]. Thus, we can still imagine what an Aristotelian mechanics would look like were it presented in mathematical language. Let us formulate it through the following laws which will simplify the comparison with Newtonian mechanics:

I law: a body not acted upon by a force stays at absolute rest,

II law: $m\mathbf{v} = \frac{1}{h}\mathbf{F}$,

III law: if a body A acts with force \mathbf{F} on a body B then B acts with force $-\mathbf{F}$ on A , where these forces have the same line of action.

Here $h > 0$ is a proportionality constant independent of the body and with the dimension of a frequency. The proportionality constant h could be omitted provided we redefined the unit of force. We shall include an additional hypothesis which is also tacitly assumed in Newtonian mechanics

Zeroth law: Every body has a mass m which is additive (extensive). The total mass of an isolated system is preserved in time.

As it happens for Newtonian mechanics the first law, rather than being a trivial consequence of the second, serves to remind us what is the kinematics of the theory, namely what is the spacetime structure. Indeed, the first law clarifies that there is a special frame, and hence that spacetime has to be regarded as a product between a real line of time \mathbb{R} , and an Euclidean space \mathbb{E}^3 . It is with respect to the Cartesian coordinates of this frame that the second law is expressed.

The main information of the second law stays in what is not explicitly stated, namely in the fact that forces belong to a vector space and hence, that they can be added as vectors. It also clarifies that, contrary to Newtonian mechanics, Aristotelian mechanics is a first order theory. This should not come as a surprise. For Aristotle the velocity of a body is proportional to the force applied on it, and inversely proportional to its mass (Physics VII 249b-250a). As it has been observed by some authors, the analog of Newton's second law for Aristotle is then Stokes' law. The acceleration phase is purely transitory and due to the fact that the applied force could be variable. Admittedly, this theory cannot explain the approximate uniform motion of a projectile

without invoking some weird mechanism to justify its slow deceleration. Indeed, in the middle ages the example of the projectile was often used by J. Buridan and other philosophers to show that Aristotelian mechanics was untenable.

We included a third law, coincident in form with that of Newtonian mechanics. It has some desirable consequences. For instance, through it it is possible to prove that the center of mass of an isolated system does not change in time. The Newtonian version states that it moves with uniform velocity but here it must correctly keep the same position in the absolute frame since by isolation no exterior force can set it in motion. Thanks to the second statement of the third law we have that the angular momentum of any isolated system vanishes. Thus, without some exterior force a rigid macroscopic body cannot neither translate nor rotate.

In what follows with *Aristotelian mechanics* we shall generically refer to a broader set of theories which we now introduce. We take for well established that a correct mechanics should be a second order theory, and that the first order version given above should be recovered ignoring the transitory acceleration phases. We replace the previous laws with the following

I' law: A body, not acted on by a force, decelerates towards its natural state of absolute rest,

II' law: $m\mathbf{a} = \mathbf{F}_f + \mathbf{F}$,

III law: if a body A acts with force \mathbf{F} on a body B then B acts with force $-\mathbf{F}$ on A , where these forces have the same line of action,

plus the Zeroth law. Here \mathbf{F}_f is a universal friction force which depends on the velocity of the body, on its mass and, in a special version that we shall explore, even on the external force \mathbf{F} . We shall assume that at least the force component parallel to the velocity, and hence responsible for the negative work, be proportional to mass

$$\mathbf{F}_f = -a_f^{\parallel} m \hat{\mathbf{v}} + \mathbf{F}_f^{\perp} \quad (1)$$

where a_f^{\parallel} might depend on velocity. Furthermore, we shall assume that in absence of other forces \mathbf{F}_f has no perpendicular component. The force to which the first and third law refer to is \mathbf{F} , since it is understood that the friction force acts on every body. The reader might assume $\mathbf{F}_f^{\perp} = 0$ on first reading since our interest in the general case will be clarified only in Sect. 5.

The above classical form of Aristotelian mechanics is then recovered for

$$\mathbf{F}_f = -hm\mathbf{v}.$$

This linear dependence on momenta is particularly useful as it shows that the second law can be assumed for imaginary point particles, the same law for macroscopic bodies being just a consequence of linearity (through the usage of the center of mass). In particular, other options for \mathbf{F}_f should come with a specification of what *body* really means in the previous laws, as the non-linearity of the theory would imply a failure of the extensive property.

We end the section remarking the coherence of Aristotelian mechanics. Indeed, it is never sufficiently stressed that there is an nice consistency between the first and second laws and the spacetime product structure $\mathbb{R} \times \mathbb{E}^3$. Only the assumption of this absolute spacetime structure allows us to make sense of the first and second law, and conversely, without a friction force the theory would reduce to Galilean mechanics and the absolute space would become unobservable and would have no clear epistemological status.

3 Reye's assumption and relativistic mass

As the above formulation clarifies, Aristotelian mechanics is nothing but Newtonian mechanics plus a friction force. Curiously, we do not have to change the spacetime structure since Newton formulated his theory on absolute space, though the lack of a universal friction force prevented its identification. In modern physics there is a way of expressing Newtonian physics without resorting to the concept of absolute space, namely using the concept of fibration over time. The reader is referred to [8, 13, 26, 32] [27, Chap. 17].

In the framework of classical mechanics let us consider the motion of a body subject to a friction force \mathbf{F}_f . Later we shall consider the introduction of an additional force \mathbf{F} . We wish to take into account the effect of friction on mass. In order to fix the ideas the reader might think of a block moving on a rough surface. We assume that the block loses mass because of wear, and that in a given time interval the lost mass (equal to the debris mass) is proportional to the work done by friction forces.

This is the Reye's hypothesis for dry friction¹ [29]. This assumption is simple and elegant because it basically says that the work done by friction forces, rather than being completely dispersed into heat, goes in a given proportion into the breaking of the molecular bonds that keep the block molecules together. Although called "assumption" or "hypothesis" this is really an experimental fact in its own domain of applicability. As we shall promote it to a universal law, we shall apply it at regimes of velocity, mass, and acceleration which go far beyond the framework of applied mechanics that originally motivated it. Mathematically, it reads

$$\dot{m} = \frac{1}{c^2} \mathbf{F}_f \cdot \mathbf{v}, \quad (\text{Reye})$$

where $\frac{1}{c^2}$ is Reye's proportionality constant where $c \in (0, +\infty]$ has the dimension of a velocity. Let F_f^\parallel and F_f^\perp be, respectively, the module of the component of the friction force parallel to \mathbf{v} , and perpendicular to \mathbf{v} . Using Eq. (1) we obtain

$$\dot{m} = -\frac{1}{c^2} a_f^\parallel p. \quad (2)$$

¹Historical note. In German and Italian University courses in applied mechanics Reye's assumption has been taught at least since the first half of the 20th century [4, 12, 35]. Though applications of this theory appeared in English [24], Reye's ideas were long ignored in the English and American literature. Similar conclusions have been reached only much later by other authors [3, 14]. Reye's assumption allows one to calculate the distribution of pressure in the contact of two surfaces and hence to extract the friction force, e.g. a rotating horizontal disc above a horizontal plane.

The idea behind this Reye's type stipulation is that the friction force arising in our model will be due to the interaction with a pervasive *vacuum*, alternatively called *reservoir*. If this law could be proved to be true then, given more information on the nature of the vacuum state, it could possibly be justified with some kind of microscopic mechanism. However, at this stage we do not try to make assumptions on the nature of this medium and take the above law as given.

It remains to write down the first cardinal equation for the motion of the body under friction forces. To fix the ideas the reader might still think at the example of the block moving on a rough horizontal surface. Since we are in presence of a variable mass system we have to use the formula

$$m\dot{\mathbf{v}} = \mathbf{F}_f - \dot{m}(\mathbf{v} - \mathbf{v}_d) \quad (3)$$

where \mathbf{v}_d is the velocity of the debris. This formula much used in rocket theory goes back to Painlevé and Seeliger (1890) and can be deduced from the conservation of momentum (see [15] for a nice account on the history of this formula). Since the debris have been detached because of their motion with respect to the vacuum (the horizontal surface in the block example) it is natural to assume that after detachment the debris do not move anymore with respect to it (i.e. they become part of the vacuum), thus $\mathbf{v}_d = \mathbf{0}$,

$$m\dot{\mathbf{v}} = \mathbf{F}_f - \dot{m}\mathbf{v} \quad (4)$$

Denoting with $\mathbf{p} := m\mathbf{v}$ the linear momentum we obtain the system of equations

$$\dot{m} = -\frac{1}{c^2} a_f^{\parallel} p, \quad (5)$$

$$\dot{\mathbf{p}} = \mathbf{F}_f = -a_f^{\parallel} m\dot{\mathbf{v}}. \quad (6)$$

These are the equations that govern the motion of a free body in our theory. We regard equation (5) as a consequence of Reye's assumption, and hence as the result of the balance between work and mass transfer. Equation (6) must instead be regarded as the consequence of an instantaneous conservation of momentum in the interaction with the vacuum state. A special solution is provided by $v = 0$, according to which mass is constant. Bodies which satisfy this condition are said to be at *absolute rest* or simply at rest.

Multiplying the first equation by m , multiplying scalarly the second equation by $\frac{1}{c^2} \mathbf{p}$, and taking the difference we obtain that there is a constant m_0 such that

$$m^2 - \frac{1}{c^2} \mathbf{p}^2 = m_0^2, \quad (7)$$

or equivalently

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad v < c \quad \text{if } m_0 > 0, \quad (8)$$

$$mc^2 = pc, \quad v = c, \quad \text{if } m_0 = 0, \quad (9)$$

$$m = \frac{|m_0|}{\sqrt{\frac{v^2}{c^2} - 1}}, \quad v > c \quad \text{if } m_0 \text{ is imaginary}, \quad (10)$$

Thus both mass and velocity are related as in Special Relativity (the latter being the tachyonic case). The time dependence is nevertheless different, as in the first bradyonic case both velocity and mass decrease while preserving the relativistic relationship. From Eq. (5) and (6) taking into account that the mass m is always positive, we obtain

$$\dot{\mathbf{v}} = (\mathbf{F}_f - \dot{m}\mathbf{v})/m = -a_f^{\parallel}(1 - \frac{v^2}{c^2})\hat{\mathbf{v}}. \quad (11)$$

This equation shows that any body with velocity c will keep moving in straight line preserving it. In other words, a body moving at this characteristic speed is insensitive to the deceleration which one would intuitively expect by friction. Of course, in the example of the block one should not expect the existence of such a special velocity as c would be very high at a range of velocities for which Reye's assumption does not hold.

Even bodies admitting the special velocity c will experience a non-trivial dependence of mass on time. From Eq. (5) we obtain

$$m(t) = m(0) e^{-\frac{1}{c} a_f^{\parallel}(c)t}, \quad \text{for } v = c. \quad (12)$$

Thus it decreases since $a_f^{\parallel} > 0$.

In the bradyonic case Eq. (11) implies that the velocity will further decrease preserving the bradyonic condition $v < c$. Given the dependence of mass on velocity (8), as the velocity goes to zero the mass m approaches m_0 . We can call m_0 the *absolute rest mass* where we added the adjective *absolute* to avoid confusion with the special relativistic interpretation. Here m_0 is the mass as measured in the absolute frame when the particle has (approximately) come to rest. So far we provided no connection with the mass as measured in a frame comoving with the body. We shall call m_0 *absolute rest mass* even for $m_0 = 0$ though in this case the terminology is improper as the body will proceed at constant speed without ever reaching a state of absolute rest.

Similar considerations prove that a body in a tachyonic status increases its velocity thus preserving that condition. The mass decreases and goes to zero according to Eq. (5).

We now consider two possible models for the parallel component of the friction force. From section 4 we shall only consider the Coulomb case as it is favored by some cosmological and astronomical data. It is also more natural given the origin of the theory, since Reye's assumption was conceived for dry friction for which the Coulomb's type force provides the best approximation. We shall also restrict most of the analysis to the case $v \leq c$.

3.1 Coulomb friction

Let us calculate the dependence of velocity on time for Coulomb friction

$$\mathbf{F}_f = -ma_p\hat{\mathbf{v}}, \quad (13)$$

where a_p is a characteristic acceleration (for the example of the block over a rough horizontal surface $a_p = \mu g$ where g is the gravitational acceleration and μ is the

dynamic friction coefficient). The subscript p stands for *parallel*. We have $a_f^\parallel = a_p$ and from Eq. (11) we get

$$\dot{\mathbf{v}} = -a_p(1 - \frac{v^2}{c^2})\hat{\mathbf{v}}. \quad (14)$$

Thus, either $v = 0$ or

$$\dot{v} = -a_p(1 - \frac{v^2}{c^2}).$$

This equation can be integrated and gives

$$\begin{aligned} v(t) &= c \tanh(\varphi - \frac{a_p}{c} t), & \text{for } 0 \leq v < c \\ v(t) &= c \tanh^{-1}(\varphi - \frac{a_p}{c} t), & \text{for } v > c. \end{aligned}$$

where φ is an integration constant (interpreted as the rapidity in the former case). We observe that in the bradyonic case, and for Coulomb friction, the time needed to reach a complete stop is finite, namely $\Delta t = c\varphi/a_p$.

The dependence of mass on time is

$$\begin{aligned} m(t) &= m_0 \cosh(\varphi - \frac{a_p}{c} t), & \text{for bradyonic mode,} \\ m(t) &= |m_0| \sinh(\varphi - \frac{a_p}{c} t), & \text{for tachyonic mode.} \end{aligned}$$

In the bradyonic case it becomes equal to the absolute rest mass when the particle reaches absolute rest. In the tachyonic case it goes to zero quite rapidly as it happens for the lightlike case.

If a body moves at speed c then its mass decreases as: $m(t) = m(0) e^{-\frac{a_p}{c} t}$.

3.2 Stokes friction

Let us calculate the dependence of velocity on time for friction forces proportional to velocity

$$\mathbf{F}_f = -h m \mathbf{v}, \quad (15)$$

where h is a constant with the dimension of a frequency. We have $a_f^\parallel = hv$ and from Eq. (11) we get that \mathbf{v} has constant direction and

$$\dot{v} = -h(1 - \frac{v^2}{c^2})v.$$

This equation can be integrated and gives

$$\frac{|1 - \frac{v}{c}|}{1 + \frac{v}{c}} (\frac{v}{c})^2 = K e^{-2ht},$$

where $K \geq 0$ is an integration constant. This equation could be inverted to find $v(t)$ but the analytic expression is not particularly illuminating. What is important is that for small velocities $v \propto e^{-ht}$ thus though the body will take infinite time to

come to rest, it will cover a finite path and hence, at any practical effect, it can be considered to come to rest in a finite time, namely when the amount of space to be covered becomes negligible with respect to the size of the body.

The dependence of mass on time can be deduced from that of the velocity. Indeed, using Eq. (6) we obtain that they are simply related through the equation $mv = K'e^{-ht}$. It is convenient to define $a_p := hc$ so a body which moves at speed c has its mass decrease as $m(t) = m(0) e^{-\frac{a_p}{c}t}$ as for Coulomb friction.

4 Limits and completion of the theory

From this section we consider only the Coulomb friction case, although a similar analysis could be performed for a_f^{\parallel} dependent on velocity. For instance, in order to deal with Stokes friction it is sufficient to replace ‘Coulomb’ with ‘Stokes’ in the next instances.

In the previous study there appeared two constants. The velocity c and the acceleration a_p . In the limit $c \rightarrow +\infty$ the previous equations show that the mass does not change, and hence the theory describes simply a body moving under Coulomb friction in the simplest mechanical case which does not introduce Reye’s hypothesis. Thus for low speed the theory reduces to Aristotelian mechanics with Coulomb friction.

We wish to show that in the limit $a_p \rightarrow 0$ the theory gives Special Relativity, or equivalently that for large accelerations the theory reduces to Special Relativity. It must be remarked that so far we have made no assumptions which relate observations made in the absolute frame with observations performed on a frame moving uniformly with respect to it. Therefore, the theory is incomplete and must be complemented with further assumptions to be contrasted with observations.

In this respect it is useful to realize that the mere symmetry of the dynamical equations with respect to some group of coordinate transformations, as for Newton laws with respect to Galilei transformations, does not imply a relativity principle. In fact, one of the physical contents of the relativity principle is precisely that of clarifying that the new coordinates are not mere artifacts, but are instead connected with measured lengths and time intervals in a different frame. Thus, the validity of a relativity principle, though permitted by the formalism, is ultimately a subject of observation.

Coming to our model, in the limit $a_p \rightarrow 0$ the friction vanishes, the natural motion for free particles becomes the uniform motion, and it becomes impossible, in our Aristotelian theory complemented with Reye’s assumption, to observe the underlying absolute space. As a consequence, the theory admits the mathematical possibility of embodying, in this limit, a relativity principle. We therefore stipulate that the theory should predict the validity of a relativity principle in the limit $a_p \rightarrow 0$. In fact, we must stipulate something more if we want to connect the observations made in different frames for $a_p \neq 0$. It is natural to embody in our theory the following weak form of the relativity principle

Every law of physics which does not involve in any non-negligible way the universal friction force \mathbf{F}_f , must be invariant for the set of (inertial)

observers which move uniformly with respect to the absolute reference frame.

Since the effects of the friction force become negligible for $a_p \rightarrow 0$, this weak relativity principle implies the usual relativity principle in that limit.

There are very few possibilities concerning the possible symmetry groups that could express the relativity principle for $a_p \rightarrow 0$. Under mild conditions on causality, parity, homogeneity and isotropy they reduce to the Galilei and Lorentz groups [19,20].

The theory developed so far naturally suggests which group should be used. Indeed, let us observe that in the limit $a_p \rightarrow 0$ equation (8)

$$m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}},$$

expressing the dependence of mass on velocity, and Eq. (7) do not change, while equations (5)-(6) reduce to the correct ones for free motion in Special Relativity.

It is known that an extensive quantity m in a theory which admits the Lorentz relativity group must depend on velocity as follows

$$m(v) = \frac{m'_0}{\sqrt{1 - v^2/c'^2}},$$

where m'_0 is the rest mass of the body (i.e. measured in its comoving frame) and c' is the constant speed of particles with $m'_0 = 0$, while it must be independent of velocity for a Galilei relativity group, i.e. $m(v) = \text{const}$ [9, 28].

In our theory with $a_p \rightarrow 0$ (think of a_p as arbitrarily small) let us consider the slow deceleration of the body which reaches absolute rest in the remote future. Our expression for $m(v)$ must coincide with that deduced from the relativity principle (the coincidence for two different values of v will suffice). In particular, we have to discard the Galilean possibility and conclude that the relativity group for $a_p \rightarrow 0$ has to be the Lorentz group and that $m_0 = m'_0$, $c = c'$. This argument proves that m_0 is not only the *absolute rest mass* in the remote infinite future but also the *rest mass* as measured in the comoving frame (at least for $a_p \rightarrow 0$).

This conclusion does not change for $a_p \neq 0$, unless we complicate our theory with the introduction of additional characteristic dimensional quantities. Indeed, the dependence of m'_0 can be expressed $m'_0 = m_0 f(a_p, m_0, c)$ where $f \rightarrow 1$ for $a_p \rightarrow 0$. For dimensional reasons the only dimensionless function is $f = 1$, and hence we conclude that $m_0 = m'_0$ even far from the limit $a_p \rightarrow 0$.

Although we know that the theory does not respect the usual relativity principle - for it suffice to wait enough to observe in which privileged frame all the bodies go at rest - we know, by our weak relativity principle, that it must satisfy a relativity principle for all phenomena that do not involve the universal friction. For this restricted set of phenomena the kinematical symmetry group is again the Lorentz group for, essentially, the same argument given above. One can obtain this result from another route, namely taking into account that Landau and Sampanthar [18] proved that if mass is extensive, conserved, depends on velocity and on rest mass as we just established, and the relativity principle holds, then the relativity group of transformation is the Lorentz group.

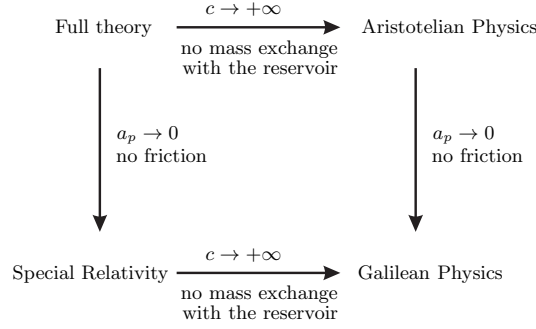


Figure 1: The proposed full theory, a Aristotelian theory under Coulomb friction and Reye's hypothesis, unifies two regimes, that of Aristotelian physics, obtained for small velocities, and that of Special Relativity, obtained for large accelerations.

We are therefore lead to the conclusion that our theory coincides with Special Relativity in all aspects that do not involve the universal friction, including time dilation and mass-energy equivalence. Mathematically, it is coincident with Special Relativity in which we added an acceleration field on the relativistic velocity space \mathbb{H}^3 - a field which takes a particular symmetric form in the privileged absolute frame. Of course, the interpretation is quite different.

We wish to stress that this natural conclusion has been obtained without using the frame invariance of the speed of particles with $m_0 = 0$. Indeed, we started from a universal friction theory and concluded that the limiting theory for $a_p \rightarrow 0$ should obey a relativity principle, and should in fact be Special Relativity with its Lorentz group symmetry, rather than classical mechanics with its Galilei group symmetry. Our argument clarifies the tight link between relativity theory and friction. In particular, it shows that the absolute space (ether) concept might directly lead to Special Relativity, a conclusion which is at odds with common knowledge.

Following the above thread of arguments there was indeed the possibility of obtaining a Galilean relativity group and hence Newtonian mechanics. We had to switch off Reye's assumption placing $\frac{1}{c} = 0$. However, since this condition is equivalent to $c \rightarrow +\infty$ in our model, we have that this possibility is merely a special case of our theory according to the usual limit by which relativistic physics reduces to classical physics for low speeds.

Finally, we wish to comment on the epistemological status of the relativistic mass concept. There have been repeated objections towards the introduction of this concept in Special Relativity on the ground that the concept of mass should not be relative to a frame. According to this school the concept of relativistic mass should be avoided in favor of the concept of energy [1, 23]. While I am partially sympathetic with these ideas, we have to admit that the concept of relativistic mass is the one that preserves the additive and extensive properties of the mass concept in classical mechanics, and which can therefore serve the intuition in the transition from classical to relativistic physics [16, 31]. Whether this intuition is really misleading is still a subject of debate. However, I remark that in the present theory the mass $m(v)$ has a completely different

and more objective epistemological status. For, if we restrict the usage of $m(v)$ with respect to the absolute frame, then the concept becomes absolute rather than relative. Clearly, the whole interpretation of our theory is based on this mass, and in this section we learned that it can be reinterpreted as energy up to the usual c^2 factor: $E = mc^2$.

Thus in our theory, the equations of conservation of mass and linear momentum for a collision e.g. (i stands for *initial* and f for *final*)

$$m_{1i} + m_{2i} = m_{1f} + m_{2f}, \quad (16)$$

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f}. \quad (17)$$

written down for the absolute frame, are such that the former can be equally well interpreted as a mass conservation equation (Zeroth law) or as a relativistic energy conservation equation.

A final word must be spent to properly account for particles with vanishing rest mass $m_0 = 0$. Equation (7) or (9) show that there is a mass (energy) indeterminacy connected with this type of particles. In order to fix this indeterminacy it is necessary to assume that some observable quantity relates with it. As it is done in Special Relativity, we identify particles moving at speed c with photons and assume Planck's formula

$$E = h_P \nu, \quad \text{if } v = c, \quad (18)$$

where ν is the frequency of the photon and h_P is the Planck constant.

5 Additional forces

Given the established equivalence between mass and energy it is convenient to rewrite Eqs. (5)-(6) through the variable $E = mc^2$ (observe that $\mathbf{p} = m\mathbf{v}$ implies $\mathbf{p}c^2 = E\mathbf{v}$).

$$\dot{E} = -a_p p, \quad (19)$$

$$\dot{\mathbf{p}} = -\frac{a_p E}{c^2} \hat{\mathbf{p}} + \mathbf{F}_f^\perp. \quad (20)$$

If we include an additional force, taking into account the effect of work on energy

$$\dot{E} = -a_p p + \mathbf{F} \cdot \mathbf{v}, \quad (21)$$

$$\dot{\mathbf{p}} = -\frac{a_p E}{c^2} \hat{\mathbf{p}} + \mathbf{F}_f^\perp + \mathbf{F}. \quad (22)$$

For a vanishing universal friction (i.e. $a_p = 0$, $F_f^\perp = 0$) these reduce to the usual formulas in Special Relativity, and imply the conservation of the scalar: $m_0^2 c^4 = E^2 - p^2 c^2$.

It must be recalled here that the force in Special Relativity is conveniently defined as $\dot{\mathbf{p}}$, because in this way the conservation of momentum implies Newton's third law, and moreover, from $E = c\sqrt{\mathbf{p}^2 + m_0^2 c^2}$ it follows $\dot{E} = \mathbf{F}_{tot} \cdot \mathbf{v}$, which is the usual kinetic energy theorem. The so called covariant world force $F^\mu := dp^\mu/d\tau$, on the contrary, is of little utility especially when considering collisions.

Equation (21) can still be interpreted as a Reye's type of relation. Indeed, it can be rewritten

$$\dot{m} = \frac{1}{c^2} \dot{L}$$

where $\dot{L} = -a_p m v + \mathbf{F} \cdot \mathbf{v}$ is the work done in the unit of time by the friction force and by the other forces.

Thus, the force \mathbf{F} so embodied in the theory respects the relativity principle (as the preservation of m_0 clarifies). This fact can be interpreted by saying that forces implemented in this way are unrelated to the friction that brakes the symmetry, but not to the vacuum and to Reye's relation. In fact Reye's relation holds for them too, pointing to the idea that all interactions are mediated by the vacuum and should not be thought as pertaining to the interacting bodies alone. This observation might be a suggestions on how to implement this theory in a field theoretical way.

Decomposed $\mathbf{F} = F^\parallel \hat{\mathbf{p}} + \mathbf{F}^\perp$, and using $\dot{\mathbf{v}} = \dot{v} \hat{\mathbf{v}} + v \dot{\hat{\mathbf{v}}} = \dot{v} \hat{\mathbf{v}} + \mathbf{a}^\perp$, we can write Eq. (22) as the system

$$\dot{p} = -\frac{a_p E}{c^2} + F^\parallel, \quad (23)$$

$$m \mathbf{a}^\perp = \mathbf{F}_f^\perp + \mathbf{F}^\perp. \quad (24)$$

Equation (23) is quite interesting because it proves that no bradyonic particle can exceed the characteristic speed c . Indeed, since m_0 stays constant, Eq. (23) holds true as long as $v < c$. This equation shows that $m(v)$ goes to infinity for $v \rightarrow c$ hence the first friction term in the right-hand side of Eq. (23) becomes greater than $F^\parallel \hat{\mathbf{p}}$ causing the velocity to stop its growth at a value v_{max} for which the right-hand side of Eq. (23) vanishes, namely

$$v_{max} = c \sqrt{1 - \left(\frac{m_0 a_p}{F^\parallel}\right)^2}, \quad \text{for } F^\parallel > m_0 a_p. \quad (25)$$

At this velocity the mass becomes constant, namely $m = F^\parallel / a_p$. Equation (25) is reminiscent of the original *second law of Aristotelian dynamics*, namely the claim that the velocity of a body is proportional to the force acting on it. Indeed, in our theory there is a monotonous increase of velocity with force, but there is no linearity because we adopted a Coulombian friction force rather than a Stokes' type force. Since $m_0 a_p / F^\parallel$ is very small, in any practical circumstance in which $F^\parallel \neq 0$, v_{max} is very large and it becomes very difficult to observe the above relation between force and velocity.

The just given argument for the preservation of the bradyonic status of particles works unaltered for impulsive forces, and hence for collisions. Curiously, in Special Relativity textbooks it is very common to find incorrect explanations for the fact that particles with positive rest mass cannot reach the speed of light. These arguments, based on the dependence of the relativistic mass on velocity, are simply fallacious as they provide a dynamical mechanism for something that cannot happen already for kinematical reasons (the very existence of the light cone and the very fact that massive particles are represented as timelike worldlines). On the contrary in our theory, the

kinematics does not demand that the light cone be invariant (though this turned out to be the case). Thus the just given dynamical explanation for the preservation of the bradyonic status is acceptable.

Equation (23) faces us with a new problem. What happens when the object is initially at rest ($\mathbf{v} = \mathbf{0}$) and a force $F^\parallel \leq a_p m$ is applied? In order to start moving, say at time $t = 0$, we must have $\dot{p}(\epsilon) > 0$ at some later instant. But the right-hand side reads $-a_p m + F^\parallel < 0$. Thus the particle does not move and moreover we must admit that since the left-hand side vanishes, also the right hand side vanishes. As $F^\parallel \neq 0$, writing the vectorial version of the equation, this is possible only if $-a_p m \hat{\mathbf{v}}$ can be different from 0 for $v = 0$. In particular it takes any value in a sphere of radius $a_p m$, as long as that value allows to solve the differential equation with the solution $\mathbf{v} = \mathbf{0}$. We are therefore led to the conclusion that it is necessary to introduce a static friction coefficient. The radius $a_p m$ correspond to the case $\mu_s = \mu_d$ but more generally it can have value $\mu_s \geq \mu_d$.

6 Comparison with observations

We shall consider the so far developed theory as a local approximation. Let us suppose that the Universe is a patchwork of possibly overlapping and interacting vacuum states. As an analogy, consider a table V_0 , a sheet of paper V_1 , and a coin moving over V_1 . The coin is the body which feels the friction of V_1 , which in turn can be in motion responding to the friction of V_0 . Of course there could be different sheets on the table and different coins over the same sheet.

We shall suppose that there could be a vacuum state V_S at the level of the solar system, which in turn behaves as a body with respect to the vacuum state V_G extending all over the Milky Way, which in turn moves over a vacuum state extending all over the local group and so on. We might consider that due to friction, two vacuum states will tend to be at relative rest, and when this happen they will form a single vacuum. We shall not consider the details of this interaction and the way by which one body ends interacting with a vacuum instead of another.

6.1 Pioneer anomaly

A first obvious consequence of our model is that free bodies with small velocities compared to c should show a deceleration of magnitude a_p in the absolute reference frame (Sect. 3.1). The effects of the universal friction should become measurable when $F^\parallel/m \sim a_p$.

Let us consider the vacuum state at the level of the solar system V_S . Any body should decelerate with an acceleration of magnitude a_p . The Pioneer spacecrafts do present an unmodeled acceleration approximately directed towards the Sun of magnitude [2]

$$a_P = (8.74 \pm 1.33) \times 10^{-10} m/s^2.$$

Unfortunately, according to a recent study, the anomalous acceleration seems to be non collinear with the spacecraft velocity [33], and can be accounted for by thermal radiation [34].

Also the friction model predicts a deceleration of the planets of the solar system, which decreases the radius of their orbits and hence implies an increase in the modulus of their velocity [25] (in the end $\dot{v} = a_p$, that is, the sign that one would naively expect gets inverted under Coulomb friction). However, this effect is quite small and can be cancelled by the decrease of the Sun mass due to solar wind [17].

6.2 Hubble law

Let us now consider the vacuum states containing our local group of galaxies and the local supercluster V_{LS} . Because of friction we must admit that most galaxies are almost at rest. Nevertheless, by a tired light effect the Hubble law still holds. Indeed, a photon sent at time t from a galaxy and received at time $t + \Delta t$ in the Milky way undergoes a redshift (see Eq. (12))

$$1 + z = \frac{E(t)}{E(t + \Delta t)} = e^{\frac{1}{c} a_p \Delta t} \simeq 1 + \frac{1}{c} a_p \Delta t.$$

If this tired light explanation for the Hubble law is correct then we must find that the observed value for

$$a_H := cH = (6.9 \pm 0.9) \times 10^{-10} m/s^2$$

(if $H = (72 \pm 8)(km/s)/Mpc$) coincides with a_p , and if the above explanation for the Pioneer anomaly is correct we must expect $a_p = a_P$ and in the end

$$a_H = a_P.$$

The above figures seem to confirm this prediction. This equality has been noticed by many authors [2], but this appears to be the simplest physical theory which accounts for it. This explanation is the more striking as it is compatible with Special Relativity.

This is a kind of tired light explanation of the Hubble law, however, contrary to usual tired light theories, it does not assume that the loss of energy is due to scattering with diffuse interstellar matter. The latter assumption would imply a modification in the direction of the photon, so that any galaxy would be seen as blurred and indefinite, contrary to observations. Our universal friction mechanism preserves the direction of the photon and so avoids this problem.

Unfortunately, tired light explanations of the Hubble law cannot account for the supernova light curves [10]. As it will be clarified in the next subsections, perhaps the Hubble law could be due to two mechanisms, namely tired light for sufficiently close cosmological objects (with the idea that they belong to the same vacuum state) and cosmic expansion for far away objects (as they belong to different vacuums states, different states diverging from each other). In other words, in the empty region between two vacuum patches there would be no friction and so the dynamics would be completely relativistic (recall that the theory becomes relativistic for $a_p \rightarrow 0$, and by Eq. (12) there is no tired light effect in that limit). There would still be a redshift effect but due to the relative motion of the vacuum states. One would have to explain why these two redshift effects, namely that due to tired light and that due to the universe expansion have the same magnitude.

6.3 Coldness of the Hubble flow

If the Hubble law is at least locally due to tired light, then it should be expected to hold for close celestial objects as well. The expanding universe theory predicts the Hubble law at length scales which are well beyond the scale of homogeneity for which a Friedmann-Robertson-Walker approximation of the cosmological metric would make sense. A well known puzzle in cosmology is the “coldness of the Hubble flow” namely the observation that the Hubble law holds at the scale of the local group (1-10Mpc) with a velocity scatter with respect to the Hubble flow which is very small (40 km/s), although at that scale the matter distribution is very clumped [5]. A local tired light model for the Hubble law accounts for this observation quite easily, for according to this explanation the local Hubble law does not depend on an expansion dynamics.

6.4 Perpendicular friction and MOND

So far we have not specified the dependence of \mathbf{F}_f^\perp on exterior data. We consider the following model

$$\mathbf{F}_f^\perp = ma_0 \beta\left(\frac{F^\perp}{ma_0}\right) \hat{\mathbf{F}}^\perp, \quad a_0 := qa_p \quad (26)$$

where q is a dimensionless number of the order of unity and $\beta : [0, +\infty) \rightarrow [0, +\infty)$ is a non-decreasing function such that $\beta(x) \sim \sqrt{x}$ for $x \ll 1$ and $\beta(x)/x \ll 1$ for $x \gg 1$. According to this stipulation the perpendicular component of the friction force depends on the applied exterior force and becomes negligible for $a_p \rightarrow 0$. The assumption that q is of the order of unity essentially means that we are not introducing any other dimensional parameter in our model.

Let us consider the motion of stars in a spiral galaxy under the assumption that there is a vacuum V_G extending all over it. Equation (24) reads

$$ma^\perp = ma_0 \beta\left(\frac{F^\perp}{ma_0}\right) + F^\perp.$$

where m is the mass of the star, a^\perp the magnitude of the component of the acceleration which is perpendicular to the velocity, and F^\perp is the magnitude of the exterior force perpendicular to the velocity. The previous equation can be inverted to give

$$ma^\perp \mu(a^\perp/a_0) = F^\perp, \quad a_0 := qa_p \quad (27)$$

where μ is a function such that $\mu(x) \rightarrow 1$ for $x \gg 1$ and $\mu(x) \sim x$ for $x \ll 1$. For instance, if $\beta = \sqrt{x}$ we have

$$\mu(x) = 1 - \frac{1}{2x}(\sqrt{1+4x} - 1). \quad (28)$$

We recognize in Eq. (27) the MOND relation [11, 22, 30], at least for what concerns the degrees of freedom perpendicular to the velocity. Fortunately, they are the most important in the derivation of MOND type phenomenology.

Indeed, let us recall how the Mond mechanism works applying the equation to the circular motion of a star around the galaxy. Its motion will be determined by Eq. (27) where

$$F^\perp = G \frac{M_G m}{r^2}.$$

For spiral galaxies $F^\perp/m \ll a_0$ thus, since we are in a MONDian regime, we have

$$m \left(\frac{v^2}{r} \right)^2 \frac{1}{a_0} = G \frac{M_G m}{r^2},$$

which implies at once

$$v^4 = \frac{G}{a_0} M_G. \quad (29)$$

which is the Tully-Fisher relation. We recall that, more generally, the Tully-Fisher relation states $L \propto v^p$ where L is the luminosity of the galaxy. Observationally the wave-band dependent exponent p stays in the range $[2.5, 5]$, and has the smallest scatter in the near infrared for which p is found to be close to 4, see [21]. Observations give

$$q a_p = a_0 = 1.2 \times 10^{-10} m/s^2$$

from which we obtain $q^{-1} \simeq 7$, thus q is of the order of unity as the consistency of our model required.

We conclude that the perpendicular component of the friction force can be chosen so as to reproduce all the important features of MOND. Thus the many predictions of this theory can find a place in our model. All that with the compatibility with Special Relativity in the limit $a_p \rightarrow 0$.

7 Conclusions

We introduced an Aristotelian theory in which mass decreases proportionally to the work done by the universal friction force on the body. We showed that mass depends on velocity as in Special Relativity and that there is a characteristic velocity c which is insensitive to friction. Bodies with velocity smaller than c decelerate till they reach a status of absolute rest. Bodies with velocity c preserve their velocity while they lose mass. We argued that in the limit of vanishing friction $a_p \rightarrow 0$ the theory becomes coincident with Special Relativity as the underlying absolute space becomes unobservable. This result shows that Special Relativity, often regarded as incompatible with the concept of absolute (or ether) frame, can actually be obtained from a more detailed study of the interaction of bodies with such a frame.

In the last sections we confronted the theory with experiment, showing that it can account for some puzzling cosmological observations which involve accelerations of the order of $10^{-10} m/s^2$. Although there are other phenomena that at present cannot be explained with this theory it seems worthwhile to investigate it further.

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